

# QED in strong fields

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## Abstract

We discuss the status of atomic physics in strong field. We focus on the problem of electron-positron lines observed in heavy-ion collisions and on QED effects, calculated in strong Coulomb fields, especially Delbrück-scattering. We discuss the similarities and differences between these effects and channeling respectively beamstrahlung. We investigated the prospects for photon-channeling, calculated channeling from first principles on the basis of the Dirac equation, and determined the rate for electron-positron pair production in the collision of two high-energy particle pulses.

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In this contribution I would like to put the channeling phenomenon in a larger context, namely the physics of strong electromagnetic fields, a subject extensively investigated in Frankfurt since many years. The common technical feature of the many different effects investigated under this label is that the interaction of (usually) electrons with the electromagnetic fields of the problem cannot be treated in perturbation theory, but has to be taken into account exactly [1].

To illustrate the diversity of this field I would like to address very shortly three topics, namely the status of the electron-positron coincidences observed at GSI for quite a number of years, some problems of QED in heavy atoms, and our contribution to the understanding of channeling and beamstrahlung.

## 1 Electron-positron production in heavy-ion collisions

In the collision of two heavy ions one can generate for a very short time an atomic system with an effective charge which is the sum of the charges of the colliding nuclei. This is possible because electrons with a wavelength

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of  $1/m_e \approx 400$  fm cannot resolve structures far below this scale. For the short time such a collision takes an electron becomes very strongly bound. Its binding energy can actually exceed twice its rest mass. This happens if the combined atomic charge is larger than roughly 173, depending on the amount of shielding by the other electrons. Fig.1 gives an impression of the relevant atomic processes in such an ‘overcritical’ heavy ion collisions. These processes

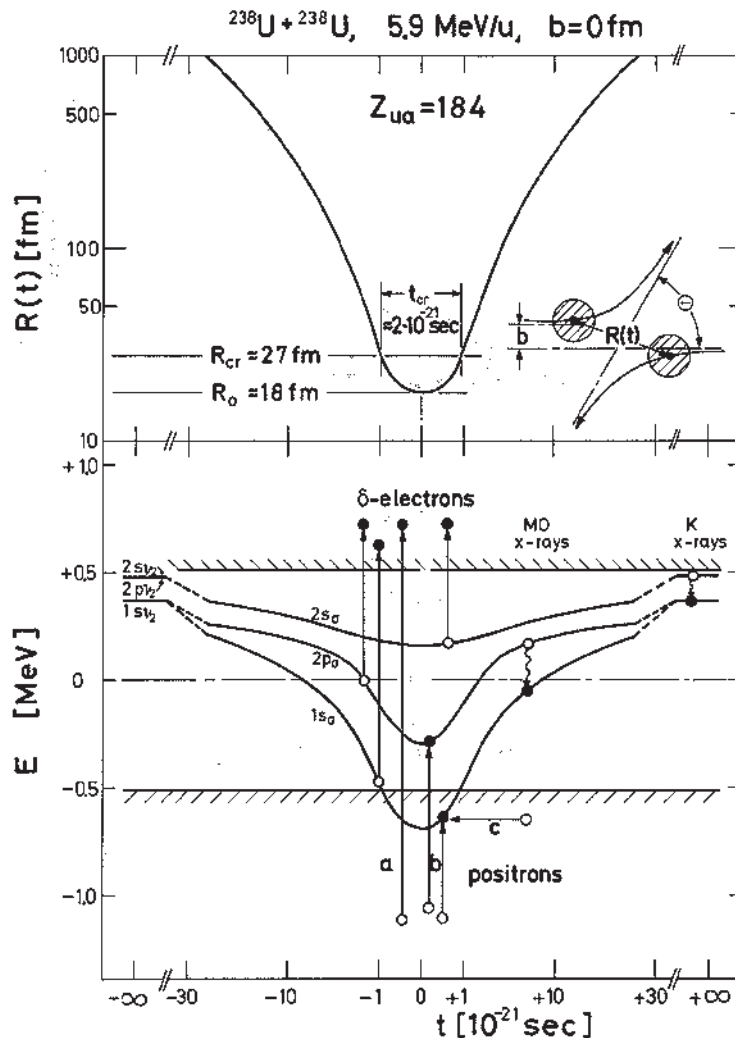


Fig. 1. The basic processes taking place during an overcritical heavy ion collision

have been extensively studied, both theoretically and experimentally. With the exception of  $\delta$ -electron production in U+Pd collisions at 6.1 MeV/u the theoretical and experimental results have been found to agree perfectly within their uncertainties, which are typically 10 - 20 percent.

The technical treatment of strong field problems is basically always the same. Because the Coulomb-potential is so strong it cannot be treated as perturbation but has to be included into the fundamental hamiltonian. For the special

case of two colliding heavy ions this reads

$$H_0(\vec{R}(t)) = \vec{\alpha} \cdot (\vec{p} - e\vec{A}) + \beta m + eV_{\text{Cb}}(\vec{R}, \vec{r}) + eV_e(\vec{R}, \vec{r}) \quad (1)$$

with

$$V_{\text{Cb}}(\vec{r}) = - \int_{V_p} d^3r_p \frac{\rho_p(\vec{r}_p)}{|\vec{r} - \vec{r}_p|} - \int_{V_t} d^3r_t \frac{\rho_t(\vec{r}_t)}{|\vec{r} - \vec{r}_t|}. \quad (2)$$

Here  $\rho$  is the nuclear charge density and the indices  $p$  and  $t$  stand for projectile and target. It is also practical to include the dominant part of the electron-electron interaction, which describes shielding of the ion charges, in the potential. Thus  $V_e$  can, e.g., be chosen as

$$V_e = \sum_k \int d^3r' \Phi_k^\dagger(\vec{r}') \frac{e}{|\vec{r} - \vec{r}'|} \Phi_k(\vec{r}') \big|_{\text{monopole}} - \frac{2e}{3} \left( \frac{81}{8\pi} \right)^{1/3} \sum_k \Phi_k^\dagger(\vec{r}) \Phi_k(\vec{r}) \big|_{\text{monopole}} \quad (3)$$

implying a Slater approximation for the exchange term. In writing down this hamiltonian we have made the monopole approximation, i.e. higher multipoles of the Coulomb-field are neglected. One can go beyond this approximation, but it was found that the contributions from higher multipoles are negligible.  $H_0$  is used to calculate the time-independent eigenstates of the two-ion system. Time dependence enters through the varying distance vector between the two ions  $\vec{R}(t)$ . The time-dependent Dirac-equation can be converted to a system of coupled differential equations by expanding the wave functions according to

$$\Psi_i(\vec{R}(t)) = \sum_j a_{ij}(t) \Phi_j(\vec{R}) \exp(-i\chi_j(t)) \quad (4)$$

$$\chi_j(t) = \int_0^t dt' \langle \Phi_j | H_0(\vec{R}(t')) | \Phi_j \rangle \quad (5)$$

$$\dot{a}_{ij}(t) = - \sum_{k \neq j} a_{ik} \langle \Phi_j | \frac{\partial}{\partial t} + iH_0(t) | \Phi_k \rangle \exp(-i(\chi_j(t) - \chi_k(t))) \quad (6)$$

These equations have to be solved for a given trajectory  $\vec{R}(t)$ . The square of the expansion amplitudes  $a_{ij}$  taken at large  $t$  leads to the occupation probability for the state  $j$ . In this way many experiments were described with high accuracy. Figure 2 and 3 are typical examples and one could show dozens of similar plots. However, the most interesting process in Figure 1, namely the

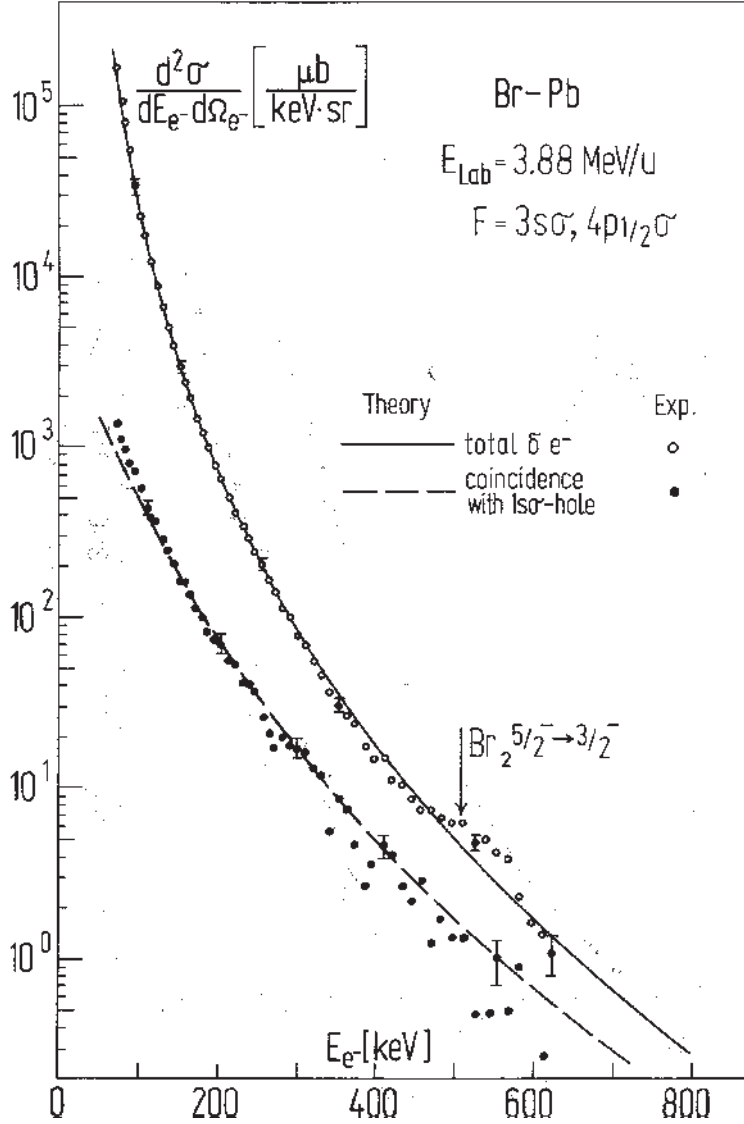


Fig. 2.  $\delta$ -electron spectra for Br on Pb. The agreement between theory and experiment is perfect, up to the well-understood contribution from an identified nuclear transition.

process c which is called ‘spontaneous pair production’ could not be observed so far and theoretical calculations show that to observe it would require the two ions to stick together much longer than usual. It is generally agreed that for the rather broad class of nuclear reactions summed over in the experiments so far, such long sticking times cannot be expected.

In spite of this fact positron experiments at GSI found pronounced positron lines [2] and even extremely sharp electron-positron coincidence lines [3]. At the time of their discovery it was tempting to interpret these lines as signal for the decay of a new particle, possibly the axion. New experiments and further theoretical investigations of many low-energy phenomena showed, however, conclusively that this is no valid possibility (for a review see [4]). We summa-

## positron spectra

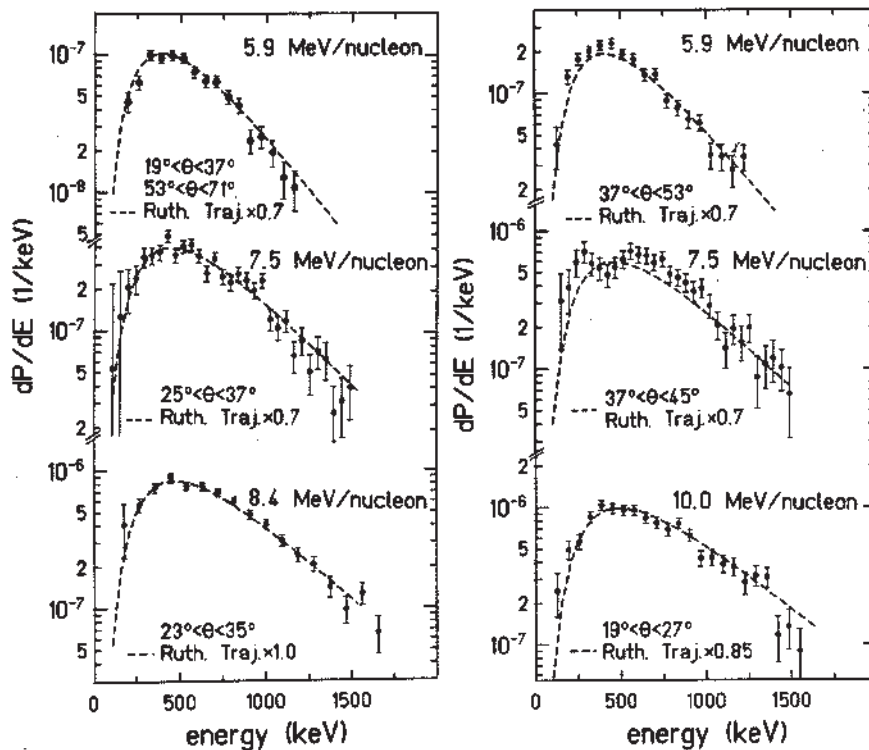


Fig. 3. Positron spectra for uranium on uranium for various projectile energies. In each case a scaling factor between 0.7 and 1.0 was introduced, which characterizes the absolute precision reached.

rized these results already early on as follows: ‘... we have to conclude that we did not find any scenario that would allow to describe the production of an X particle with the required properties without very unnatural assumptions.’ (1986, [5]) and ‘... Therefore, it is tempting to conjecture that some of the experiments are unreliable.’ (1988, [6]).

The present situation of these lines is the following: Recent EPOS [7] and Orange [8] experiments at GSI could not reproduce the line observed early although the detectors were improved to give much better statistics (especially for EPOS). A completely independent experiment at Argonne National Laboratory called APEX did not find any indication for lines [9]. At the same time it became clear that nuclear pair-conversion was less well understood than assumed, such that at least some of the data from recent years might be due to conversion, a possibility which is still excluded for the early much larger line structure. It is unclear, how this has to be interpreted, but definitely there is presently no compelling evidence for any process more exotic than nuclear pair conversion.

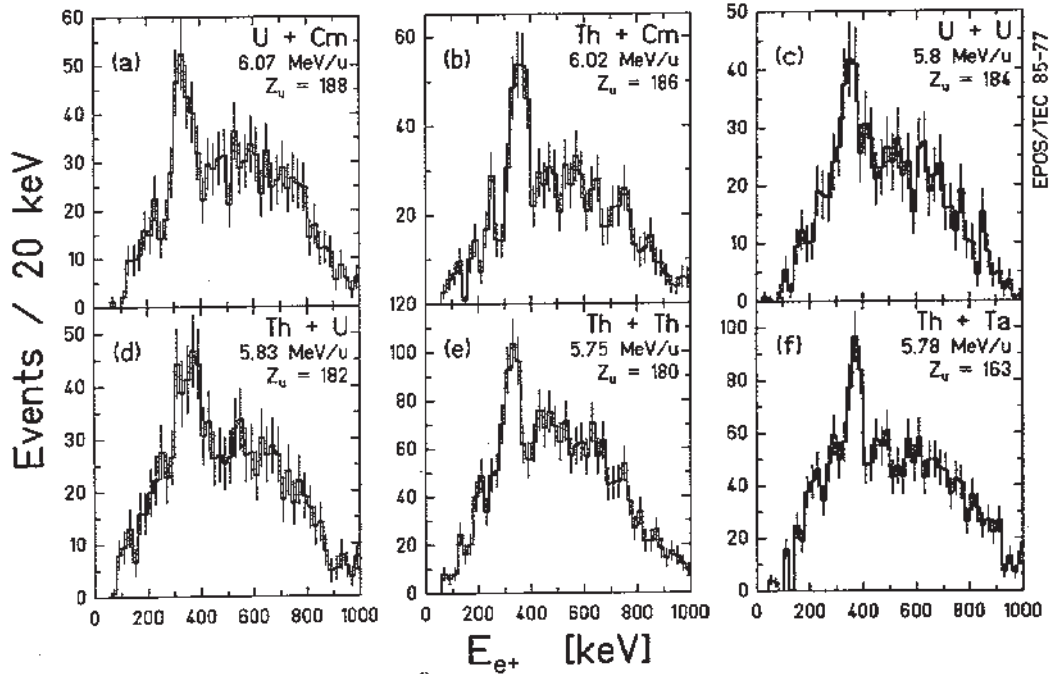


Fig. 4. The surprisingly large positron singles lines observed by EPOS in 1985

## 2 QED in strong fields

A very interesting possibility opened by high- $Z$  atoms is to test QED in a non-perturbative regime. QED is a divergent field theory, and the three basic divergences: vacuum polarization, self energy, and vertex correction have to be renormalized. This renormalization procedure in a Coulomb-potential differs from that in free space, which results in finite corrections to basically all physical quantities. The complete understanding of these corrections is indispensable for a reliable description of processes taking place in external fields. In Frankfurt we have a long tradition in calculating these [1] and presently a number of projects are under work. Let me just make a few comments on one of them, which is related to our work on channeling.

*Delbrück scattering*, i.e. the scattering of a photon off the Coulomb-field of an atom (see figure 6) was carefully measured already some time ago [10]. Systematic deviations from the lowest-order theoretical prediction were found for high- $Z$  atoms, indicating that for these the higher order terms, proportional to  $(Z\alpha)^{2n}$ , are important. A direct perturbative calculation of the next order graph is extremely involved and anyway one would like to calculate this graph exactly in all orders of  $Z\alpha$ . This is in fact possible if the perturbative propagator is substituted by the exact propagator for the Dirac-operator including the Coulomb-field

$$(i\gamma_\mu(i\partial^\mu - eV_{Cb}(r))\delta_{\mu 0} - m)S(x) = i\delta^4(x) \quad (7)$$

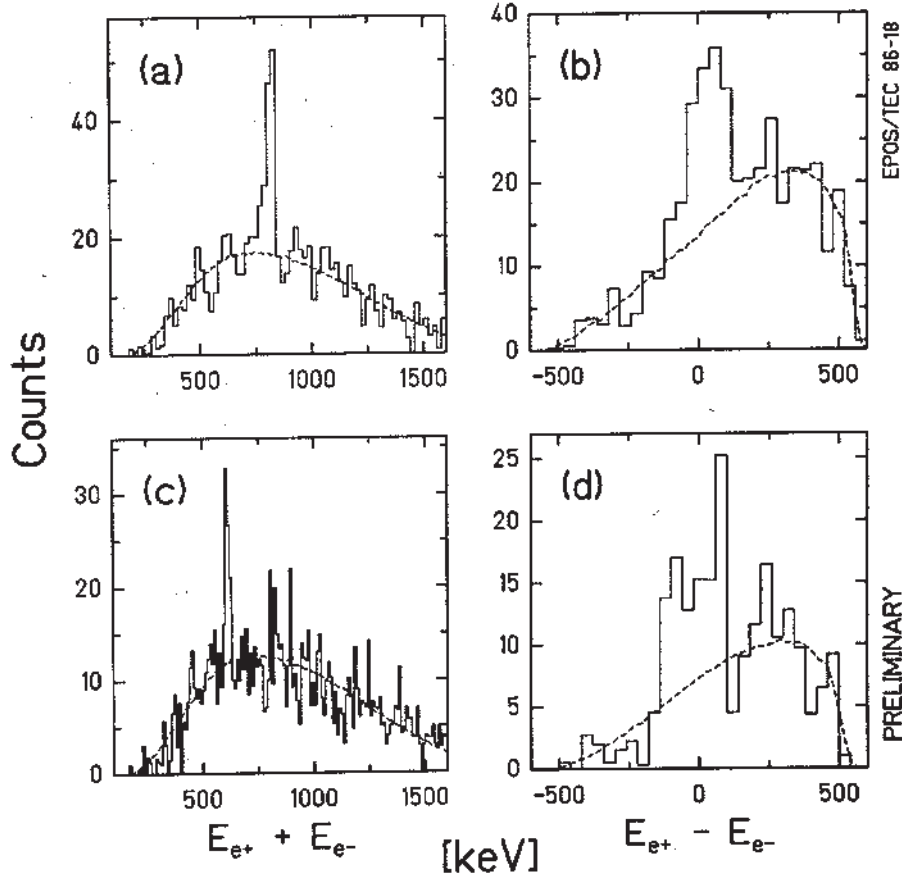


Fig. 5. The electron-positron coincidence lines observed by EPOS in 1988

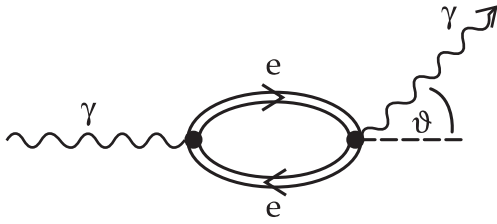


Fig. 6. The basic graph of Delbrück-scattering. The double lines represent exact propagators in the external field.

We decomposed the propagator again into contributions of definite angular momentum and inserted these into the calculation of the vacuum polarization graph. The resulting calculation is very demanding, involving thousands of angular momentum states, and requiring many numerical and analytic tricks to ensure convergence. Details can be found in ref. [11]. Some results are given in the table for photons with the energy  $\omega = 1.5 m_e$  for the scattering angle  $\Theta = 1^\circ$ . Obviously one finds corrections with respect to the lowest-order result for sufficiently large  $Z$ . These investigations are still continued to get complete systematic results for all photon energies and scattering angles.

Table 1

Delbrück scattering in all orders of  $Z\alpha$ 

$\omega$	$\Theta$	$Z$	$M_{\text{Born}}$	$M$
$1.5 M_e$	$1^\circ$	10	$2.4 \cdot 10^{-3}$	$2.39(3) \cdot 10^{-3}$
$1.5 M_e$	$1^\circ$	20	$9.6 \cdot 10^{-3}$	$9.62(8) \cdot 10^{-3}$
$1.5 M_e$	$1^\circ$	60	$8.6 \cdot 10^{-2}$	$8.9(1) \cdot 10^{-2}$
$1.5 M_e$	$1^\circ$	80	$1.5 \cdot 10^{-1}$	$1.63(2) \cdot 10^{-1}$

In another investigation we treated *channeling* by solving the Dirac equation, starting from first principles. Again this calculation was extremely involved. However, it constitutes the first completely relativistic calculation for this problem. If the physics of channeling were qualitatively similar to that of strong fields this should show up in specific relativistic effects. In fact we got rather satisfactory results (see figure 7 and 8, and [12,13] ), but we found

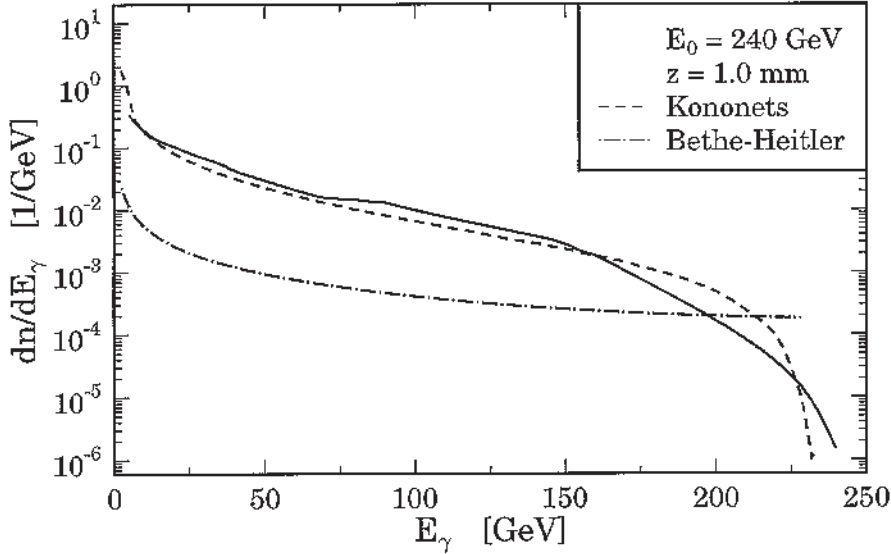


Fig. 7. Typical photon spectrum for relativistic channeling along the  $\langle 110 \rangle$  axes of Germanium, compared with the quasi-classical results of Kononets (ref. 12) and the Bethe-Heitler formula. Obviously we confirm the Kononets results. The lack of smoothness of our result is due to the finite number of states and transition matrix elements taken into account.

no specifically relativistic effects. It is usually argued that the relevant mass is very large  $m_{\perp} = \sqrt{p_z^2 + m_e^2}$ , such that one can perform quasi-classical calculations. Our results confirm this and show at the same time that for such effectively very heavy particles no phenomena occur which would be similar to overcritical binding in heavy atoms. As a by-product of this calculation we obtained the distribution of the radial distance from the channeling axes at



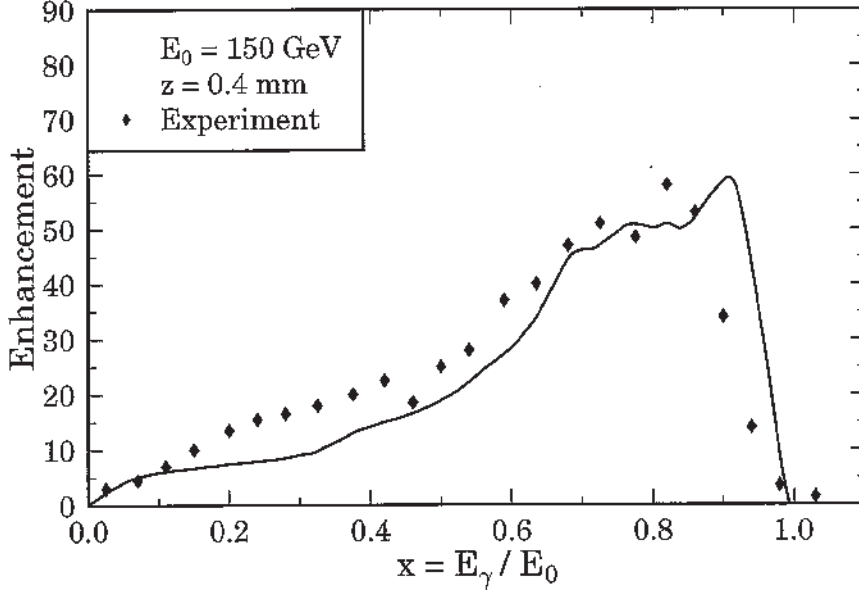


Fig. 8. Our result for the enhancement of fractional energy loss during channeling in germanium, relative to an amorphous target, compared with the data from ref. 13.

which photons are emitted (see figure 9). Again details can be found in our publication [14].

A similar conclusion was also reached by the following investigation. We studied again the Delbrück graph from figure 6, but this time for the axially symmetric field of a channeling axis. The calculation was much simplified by the fact that this problem is two-dimensional rather than three-dimensional. The physical motivation was that a photon couples also to the channeling fields due to virtual production of electron-positron pairs, and this interaction could lead to a channeling of photons with a very small angle relative to the channeling axis, see [15]. One result is shown in figure 10, where the average rotation angle around the channeling axis during the passage of a photon through the central part of the channeling potential is shown as a function of the angle of incidence  $\psi$ .  $\Phi = \pi$  would imply complete channeling, i.e. the photon would be bound to the channel. Obviously we are very far from this point, even for very small angles. For details see [15]. Again, no substantial increase of the effect with increasing photon energy was observed. Such an increase would have indicated a prolonged existence of the virtual electron-positron pair and thus a stronger interaction with the channeling potential) was observed.

Cutting a long discussion short, we would like to argue that the channeling effect is basically different from the effects in over-critical Coulomb-fields for the following reason: The fundamental process underlying all effects is the separation of an electron-positron pair in a strongly space-dependent potential. The energy gained by this charge separation is a measure for the size of the novel effects up to the point where it allows the electron and positron to become real,

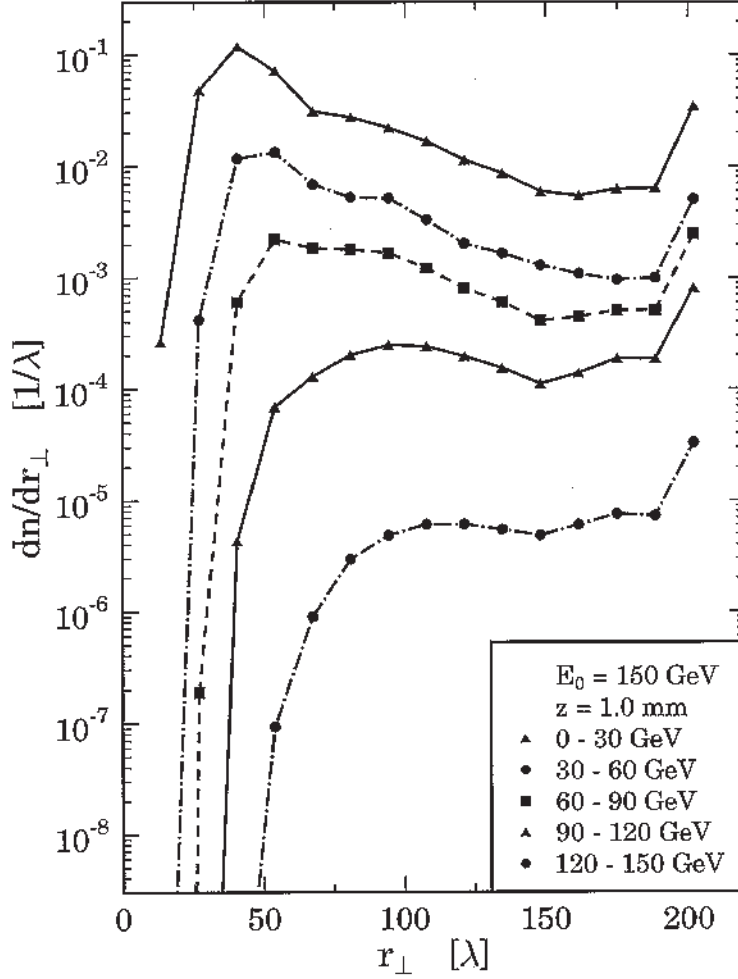


Fig. 9. Distribution of the emission points of photons as a function of the radial distance from the channeling axis.

which would be spontaneous pair production. In the channeling situation the electric fields becomes, however, only strong due to a Lorentz boost (into the rest system of, e.g., the electron). The same boost introduces also an magnetic field which confines the electron and positron to a radial distance of the size of a Landau orbit  $\Delta r \sim 1/B$ . Thus the crucial quantity (electric field strength)  $\times$  (radial separation of electron and positron) is proportional to  $E/B$ , which is basically independent of the electron velocity, i.e. the Lorentz factor  $\gamma$ . Thus no typical overcritical field effects can be expected.

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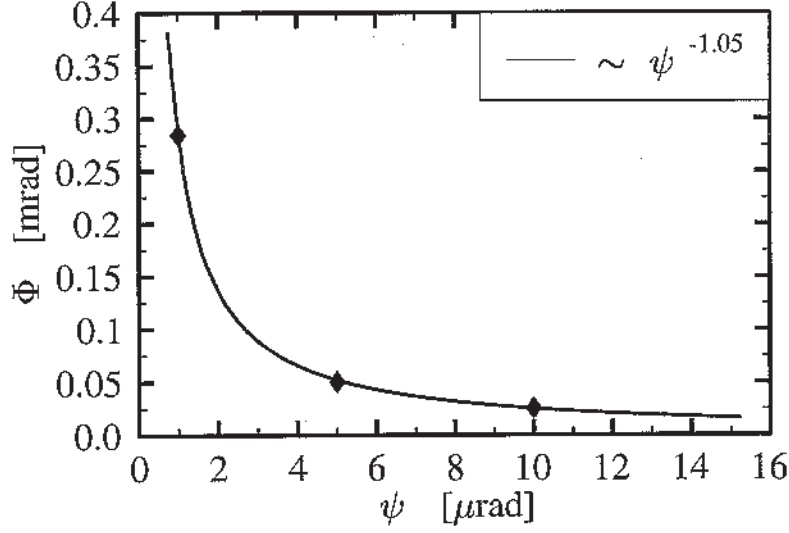


Fig. 10. The averaged angle by which a photon is scattered while passing through a channel as function of the angle of incidence relative to its axis.

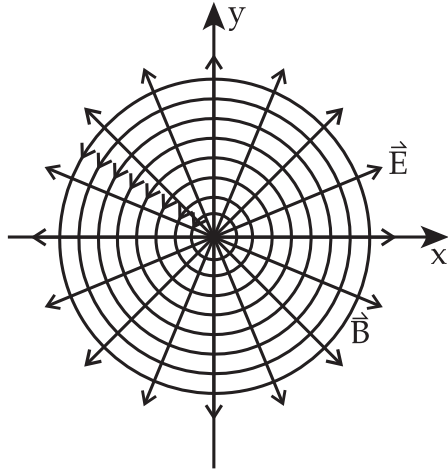


Fig. 11. The boosted electric and magnetic fields seen by a channeled particle.

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